

Design and Evaluation of Digital Resources to Enhance Creative Mathematical Thinking in a Biomathematics Context

Mohamed El-Demerdash, Christian Mercat, Jana Trgalova, Nataly Essonnier and Pedro Lealdino Filho

Abstract—In this article, we present the design and the *a priori* evaluation of digital resources, carried out within the framework of the MC Squared project (<http://mc2-project.eu>). These resources are the so-called “c-book units” (“c” for creative), which are digital books produced within a socio-technological environment allowing meshing narratives with interconnected, interactive and dynamic digital artefacts (widgets), to promote Creative Mathematical Thinking (CMT). We have developed a methodology to support creative design and the *a priori* evaluation of the CMT potential of the c-book units’ affordances. We illustrate here the design and evaluation of a c-book unit within a biomathematics context aimed at supporting learning and mathematical reasoning, using a wide range of digital widgets. We discuss the design choices resulting in the resource affordances to promote mathematical creativity in terms of personalized non-linear path, constructivist approach, and meta-cognition based activities, among others. We present experts’ *a priori* evaluation of the resource CMT potential.

Index Terms—Creativity, mathematical creativity, mathematics education, technology-enhanced learning.

I. INTRODUCTION

PROMOTING Creative Mathematical Thinking (CMT) to enable innovation is a central goal of the European Union [1]. CMT is a highly valued asset in industry and addresses current and future economic challenges. It is seen as an individual and collective construction of mathematical meanings, norms and uses in novel and useful ways [2], [3], which can be of relevance to a larger (academic, learning, professional, or other) community.

New exploratory and expressive digital media created by new designers are providing users with access to and potential for engagement with CMT in unprecedented ways taking into account the co-evolution of teaching and technology [4] or the constructionist approach [5].

The MC Squared project (<http://mc2-project.eu/>) aims at

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The authors are with S2HEP EA4148 Université Claude Bernard – Lyon 1, France. M. El-Demerdash is furthermore with Menoufia University, Egypt (corresponding author e-mail: m_eldemerdash70@yahoo.com).

designing and developing an intelligent computational environment, called C-book technology, to support stakeholders from creative industries involved in the production of media content for educational purposes to engage in collective forms of creative design of appropriate digital media. The C-book technology provides an authoring dynamic environment extending e-book technologies to include diverse dynamic widgets, an authoring data analytics engine and a tool supporting asynchronous collaborative design of educational resources, called “c-books”. The specificity of this tool lies in the tight integration and possible cross-communication between different types of widgets coming from existing standalone software using an open Application Protocol Interface (API)¹, allowing for intelligent adaptive feedback and instance storage (the user comes back to the activity in the actual learning state s/he left it). The project focuses on studying processes of collaborative design of digital media intended to enhance CMT. The article then has twofold objectives. First, it highlights, through the development of “Math for Biology” c-book unit, the design choices to foster CMT in its users. Second, it enlightens, through experts’ *a priori* evaluation, the CMT affordances of this c-book unit.

II. CREATIVE MATHEMATICAL THINKING

Based on a literature review and prior studies led by researchers involved in the project related to studying creativity and mathematical creativity [6-11] a definition that reflects our vision of CMT has been adopted which defines it as a directed intellectual activity generating novel mathematical ideas or responses over the known or familiar ones in a non-routine mathematical situation, relatively to a pedagogical context. Drawing on Guilford’ model [12], the generation of new ideas shows the abilities of fluency, flexibility, originality/novelty, and elaboration that are defined as follows:

- **Fluency** means the individual’s ability to pose or come up with many mathematical ideas or configurations related to a mathematical problem or situation in a short time.
- **Flexibility** refers to the individual’s ability to vary the approach or suggest a variety of different methods toward a mathematical problem or situation.

¹ Software candidate for html-5 integration shall follow the Application Protocol Interface specification described on <http://mc2-project.eu>

- **Originality** means the individual's ability to try novel or unique approaches toward a mathematical problem or situation.

- **Elaboration** is the individual's ability to redefine a single mathematical problem or situation to create others, by changing one or more aspects by substituting, combining, adapting, altering, expanding, eliminating, rearranging, or reversing and then speculating on how this single change would have a ripple effect on other aspects of the problem or the situation at hand.

III. THE DESIGN OF "MATH FOR BIOLOGY" C-BOOK UNIT

"Math for Biology" c-book unit² proposes a sequence of activities aiming at supporting learning and mathematical reasoning, using a wide range of widgets useful in biology such as EpsilonWriter (Dynamic Algebra System - DAS), Cinderella and GeoGebra (Dynamic Geometry System - DGS). Cross-widget communication³ allows the user to send messages such as geometric or algebraic information between EpsilonWriter and Cinderella. Some open-ended activities are designed to train a high degree of CMT such as reflecting on models, modelling spirals and code tweaking in a half-baked logo microworld. Such a tight integration among these software factories would not be possible in existing design frameworks: the open API allowed to insert instances of these software on the same page so that the user doesn't have to switch among these software, and their common state is saved in a bundled live document, but more importantly, the widgets communicate so that the work in one widget influences the other ones, especially in intelligent feedback aimed at unblocking students learning. The analysis of the usage of the c-book is as well eased by automatic logging of activities, tailored to specific requests from teachers in a much more detailed and integrated way than in usual Learning Management Systems (LMS), because usage is traced to the level of individual widgets.

The c-book unit consists of four sections interrelated through a narration about a children visiting a Fauna and Flora park. Section 1 introduces biomathematics, its meaning and its study fields and ends up with giving users the opportunity to pick and choose among different activities on biomathematics proposed in sections 2 - 4 according to their knowledge and interest. Section 2, demonstrates the need of mathematical approximation as a tool to model biological traits, and enables to reflect on magnitudes. In section 3, golden ratio is a pretext search, for doing mathematics by looking at its occurrences in nature, paintings, architecture and Fibonacci numbers. The c-book unit ends with section 4, where users are exposed to different kinds of spirals in nature. Users have the opportunity to upload their pictures of spirals (Fig. 1) and try out some constructs that might fit them, using different digital widgets either based on algorithms or algebra. Finally, they can build their own spiral-based artwork reminding of Celtic art.

² <http://6.mc2dme.appspot.com:80/dwo/dwo.jsp?profile=78&language=en&courseViewNr=5723809264959488>

³ Cross widget communication is a way to allow to send messages between widgets

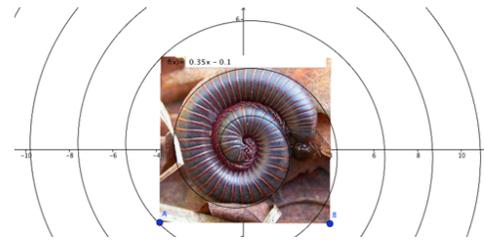


Fig. 1. A DGS widget enabling students to upload their own spiral pictures and try out some formulae that fit them.

IV. DESIGN CHOICES AND RATIONALE

The designers in the project share a common interest of producing digital resources promoting CMT. The French Community of Interest (CoI) [12] designing "Math for biology" c-book unit is a heterogeneous group gathering mathematics teachers, teacher educators, computer scientists and researchers in mathematics education.

A. CMT affordances

The designers focused on supporting the development of the CMT cognitive components (fluency, flexibility, originality and elaboration), social and affective aspects on the user's side through a relevant use of specific widgets. The C-book technology boasts affordances that can promote CMT. To take a technical metaphor, a door has a specific use; a knob is an affordance that allows for an acceptable use of the door, allowing to open and close it. Similarly, the activities in our c-book unit engage the users in compelling problems and the widgets that we setup allow for creative ways to think about them, investigate and solve them. The c-book unit opens up as well an open space for reflection and problem posing. The description of three instances of CMT affordances follow in Section E.

B. Storytelling Approach and Personalized Non-linear Path

Throughout all the activities of this c-book unit, the designers use a storytelling approach supported by interactive widgets, videos and pictures to motivate and engage users and guarantee unity of the c-book activities. After introducing the meaning of the term biomathematics and its fields of study in section 1, users are free to choose their path of learning. Therefore, the structure of the c-book unit allows users to autonomously navigate according to their interest and knowledge.

C. Constructivist Learning Approach

The c-book unit activities in sections 2 - 4 are developed based on the constructivist learning theory practices enabling students to create new experiences and link them to their prior cognitive structure, supported with learning opportunities for conjecturing, exploration, and mathematics communication.

D. Metacognition Activities

All c-book sections terminate with a meta-cognitive activity that has been designed to encourage students to reflect about their learning and enable them to further understand, analyze and control their own cognitive processes. These activities

have also been designed to develop students' written mathematical communication skills through the use of EpsilonChat, a widget for communicating mathematics in which users can easily write mathematical contents.

E. Technology Added-value

Three instances of designers' use of C-book technology are presented to highlight the technological added-value provided by such a system:

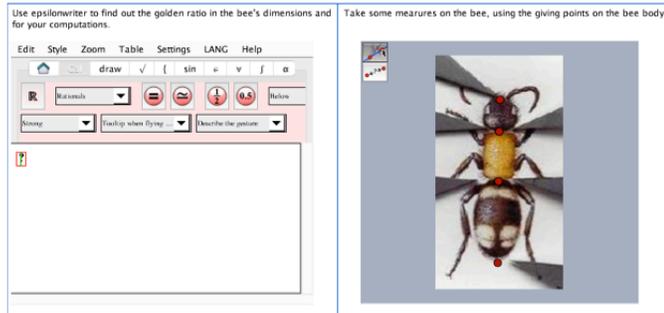


Fig. 2. Cross-widget communication to allow sending messages between Cinderella and EpsilonWriter.

The cross widget communication between Cinderella and EpsilonWriter, two widget factories⁴, enables to check whether the golden ratio models the dimensions of a bee (Fig. 2). The idea behind the cross-widget communication is to send the measurements from Cinderella widget to EpsilonWriter where users can write text and mathematical content, as well as doing calculations, in particular comparing the ratio of values. The digital golden ratio compass tool is a DGS widget enabling users to discover proportions seemingly like golden ratios in other contexts such as in insects, and in plants, artwork, etc. Users look for a picture of an object, upload it and check with the digital compass whether its dimensions satisfy the golden ratio. They can share their thoughts and findings via EpsilonChat and EpsilonWriter. The main added value here is meshing the three interconnected widgets in the same educational environment so students work on a single document which stores all their activities in their present state, a feature not presently available in other educational environments.

The C-book technology allows using external materials that couldn't be provided by the environment but are reachable by external URL links. For instance, a 3D-GeoGebra widget that enables users to change the dimensions of a cone and a cylinder in order to get the same (See: <https://tube.geogebra.org/m/2552113>).

V. THE EVALUATION OF "MATH FOR BIOLOGY" C-BOOK UNIT CMT AFFORDANCES

In this section, we present the methodology for evaluating a priori CMT affordances in pedagogical resources and present the results of the evaluation of "Math for Biology" c-book unit.

A. The Evaluation Question

We evaluate the degree to which the four cognitive components of CMT (fluency, flexibility, originality and elaboration), and social and affective aspects have been integrated and promoted through the design of the c-book unit, as perceived by the evaluators.

B. Evaluation Methodology

The evaluation of the CMT affordances is based on an evaluation tool called "CMT Affordances Grid" (See Appendix A), which has been elaborated and refined during a 3-cycle process of MC Squared project. The grid is divided into three sections. The 13 first items evaluate the c-book unit affordances towards the development of CMT in users/students. These items address the c-book unit affordances such as nature of the activities or variety of representations of mathematical concepts at stake and ask the evaluators to what extent these affordances are likely to enhance the user's cognitive processes (fluency, flexibility, originality, elaboration). The second and third sections are dealing with the social and affective aspects of the c-book unit that are likely to enhance CMT in c-book unit users. As for the first aspect, the responders were asked to evaluate the item in relation to each one of the four cognitive components of CMT in a scale from 1 (weak affordance) up to 4 (strong affordance). There was additionally an extra option called N/A in case the affordance was not applicable for the specific item. The evaluation of the CMT affordances of this c-book unit was done by three members of the French CoI, not involved in its design. It was organized in three steps. First, the evaluators had to use the c-book unit and be acquainted with the affordances. Second, a teleconference was organized by the main designer of the c-book unit to address evaluators needs for understanding and clarification. Third, the evaluators evaluated the c-book unit affordances based on the grid using an online form⁵ prepared for this purpose.

C. Evaluation Results

The chart, shown in Fig. 3, represents the evaluation of the cognitive components of CMT from the evaluators' point of view. For each of the 13 questionnaire items, the height of a color represents the proportion of the associated CMT component, while the thickness represents the mean between the four aspects for each item (the mean of means, the thicker the better). This representation is debatable since creativity is known not to foster consensus, therefore we don't expect inter-rater evaluators agreement and using the mean is blurring this effect. Indeed, the Cohen's Kappa value of 0.3 means low agreement between the evaluators: because of their different expertise, some evaluators might foresee specific creative usage where some others won't. From the evaluators' point of view, two items, Item 12: The c-book unit includes non-standard problems calling for mathematical solutions and item 13: The c-book unit includes half-baked constructs that call for intervention of the teacher were assigned the N/A value (no affordance).

⁴ Widget factory is a software system (often developed independently of the c-book environment over many years) allowing to produce sophisticated parameterizable c-book-widgets easily.

⁵ See https://docs.google.com/forms/d/1xb-wDHLVG4w2bqMFfj6h0t-6w_GCLhQIXIPD7VZAGAo/viewform

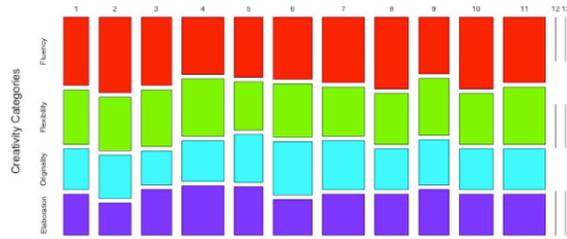


Fig. 3. Evaluation of the four cognitive components of CMT from the evaluators' point of view on the 13 items of the grid.

In Table I, we present the qualitative data for each component computing the mean from *No Affordance* (Scored 1) to *Strong Affordance* (Scored 4).

TABLE I
CMT EVALUATION SUMMARY

Fluency	Flexibility	Originality	Elaboration	Social	Affective
3.03	2.46	2.11	1.92	2.83	3.33

The highest value for this c-book in terms of cognitive aspects was fluency for which the value achieved the rank of "good affordance". It means that, in general, the c-book unit could boost the students' development of their ability to provide many responses or to come up with *many* strategies to solve a mathematical problem or challenge. Elaboration and originality are the components with lower values of weak affordance. The radar chart (Fig. 4) shows the distribution of the evaluation among the evaluated categories. This chart shows which component of CMT is most likely to be enhanced by the use of the c-book unit. In the case of "Math for Biology", it is the affective aspect, followed by fluency and social factors.

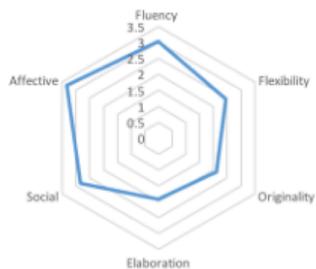


Fig. 4. Radar chart of the distribution of the evaluation among different aspects of CMT.

In line with findings reported in the literature [6-10], in Table II, collating the 13 questionnaire items, correlations among the four cognitive components of CMT are strong between some cognitive aspects.

TABLE II
CORRELATION VALUES OF CMT COMPONENTS

	Fluency	Flexibility	Originality	Elaboration
Fluency	1.00	0.89	0.67	0.75
Flexibility	0.89	1.00	0.69	0.90
Originality	0.67	0.69	1.00	0.60
Elaboration	0.75	0.90	0.60	1.00

It means that, considering a significant value of $r > 0.70$ ($p = 0.05$), we may conclude that fluency, flexibility and elaboration can be fostered at the same time. In the case of originality, there is no statistical evidence that supports the hypothesis that this component can be fostered by the other

ones. Moreover, experts did not comment that much on their quantitative evaluation. Though, they raised the need for a good orchestration to achieve the goals intended by the c-book unit.

VI. CONCLUSION

This article presents a methodology for CMT supported design and evaluation of a pedagogical product. It is illustrated on the "Math for Biology" c-book unit, an interdisciplinary digital resource within the framework of the MC Squared project. The c-book unit was well evaluated in terms of fluency and affective aspects. The c-book unit calls students to make relations with proportionality and golden ratio, emphasizing the role of mathematics and how it can be perceived in nature.

We limited ourselves to an *a priori* evaluation by experts of the resource CMT affordances. Tests with students will be conducted to validate the evaluation results.

APPENDIX

Appendix A: [CMT Affordances Grid.](#)

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